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Accelerator & Fusion Research Division

Presented at the 8th Topical Conference on
Radio Frequency Power in Plasma,
Irvine, CA, May 1-3, 1989

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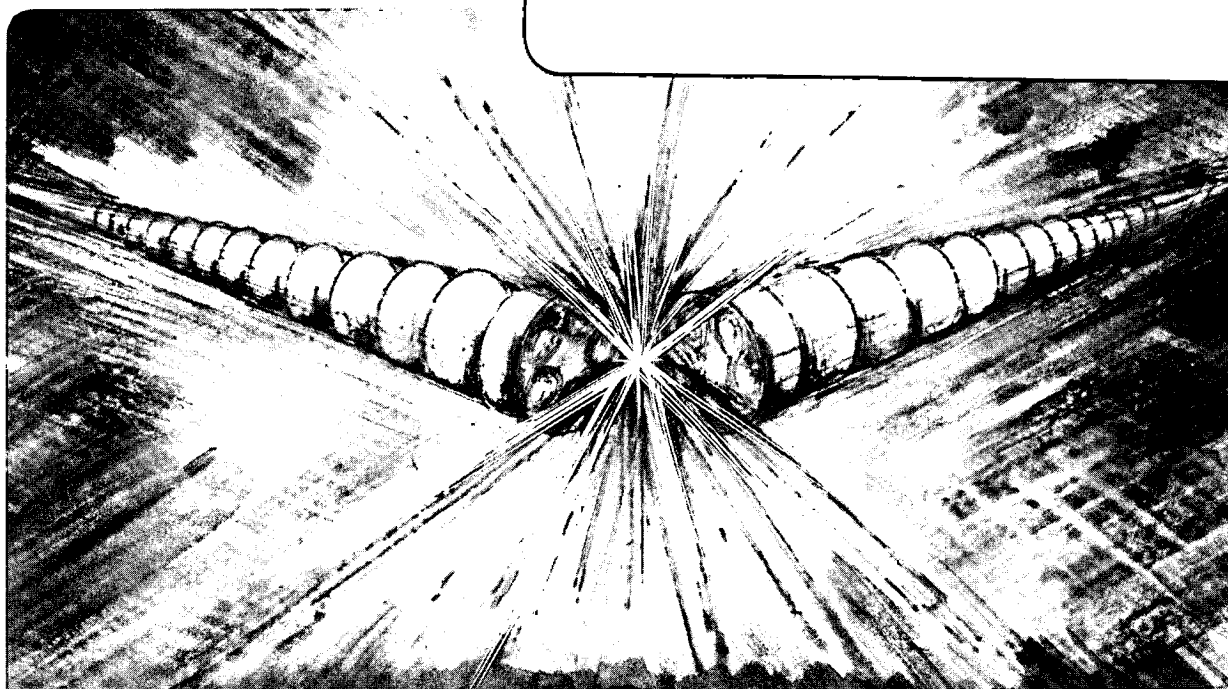
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Power in Plasma

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Work supported by the U.S. DOE under contract No. DE-AC03-76SF00098 and by U.S. DOE Magnetic Fusion Science Fellowship, administered through Oak Ridge Associated Universities.

ANALYTIC THEORY OF ICRF MINORITY HEATING*

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ABSTRACT

We present a one-dimensional analytic theory of the ICRF gyroresonant absorption and mode-conversion, for the problem of minority fundamental resonance. Using the wave phase-space method, and the theory of linear mode conversion therein, we obtain explicit expressions for the coefficients of transmission (T), reflection (R), conversion (C), absorption (A).

Ion cyclotron resonant heating is one the two main methods in tokamak heating today. Both the majority (denoted M) second-harmonic and minority (denoted m) fundamental resonances are employed, but the minority heating appears to be predominant in present-day experiments. The majority second-harmonic heating problem has been solved analytically [1], using the phase-space method and the theory of linear mode conversion, for the one-dimensional slab model. In this paper we extend our method to treat the fundamental resonance of the minority ion species.

First we discuss briefly the phase-space concept and the theory of linear mode-conversion. Consider the wave equation: $D(x, k \rightarrow -i\partial)E(x) = 0$. To the lowest order in WKB approximation, it leads to the familiar Hamiltonian equations for rays: $dx/dt = \partial\omega/\partial k$, $dk/dt = -\partial\omega/\partial x$, where $\omega(x, k; t)$ is the solution of $D(x, t; k, \omega) = 0$. Conventionally the above wave equation is regarded as a differential equation in (x, t) . However, in order to fully understand the wave dynamics, it is essential to treat x and k on an equal footing. There are many advantages to this phase-space point of view. For instance, one would never run into caustic singularities [2]. Another advantage, which is more important for our present work, is that we may have a k -independent frequency function ω , corresponding to a mode that travels only in k -space [3]; thus the physics would be obscured if we insist on staying in x -space. When two dispersion surfaces cross in phase-space, where the two modes have the same frequency and wave vector, and thus linear mode-coupling will occur. One mode, traveling on its own dispersion surface, transfers part of its energy to the other mode, which goes off on the other dispersion surface [Figure 1]. This is called *linear mode conversion*. When the coupling is localized in phase-space, the mode-conversion problem can be solved analytically [4].

The main idea in our analysis of ion gyroresonant absorption is to interpret it as mode conversions. We observe that the N^{th} harmonic gyroresonance conditions $\omega = k_{\parallel}v_{\parallel} + N\Omega(x)$ ($N = 1$ for fundamental resonance) are in fact the dispersion relations of Case-van Kampen (CVK) modes, one for each value of v_{\parallel} . Therefore

*Work supported by US DOE under contract No. DE-AC03-76SF00098

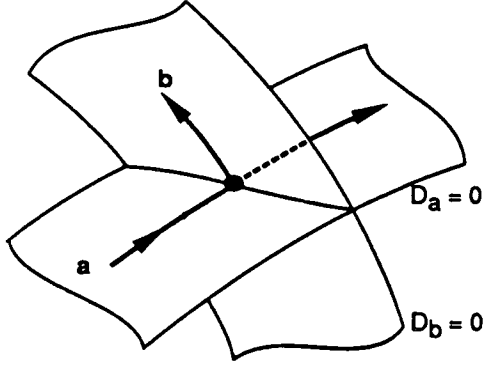


Figure 1. Linear mode conversion.

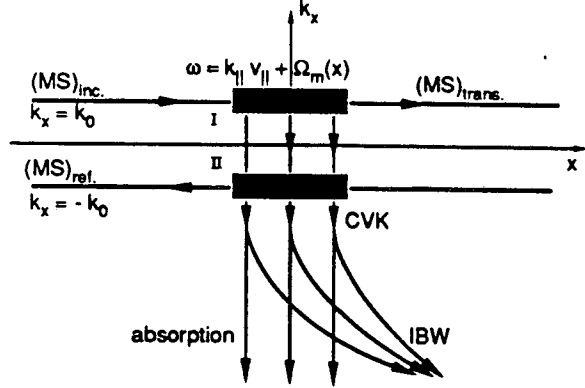


Figure 2. Dispersion diagram for ion gyroresonance.

ICRF heating can be viewed as mode conversions from the magnetosonic (*MS*) wave to a continuum of *CVK* modes [1]. Note that these *CVK* modes travel in *k*-space only, with velocity $dk/dt = -N\nabla\Omega$.

We shall study the problem in the one-dimensional slab model: $\mathbf{B}(\mathbf{x}) = \hat{\mathbf{z}}B(x)$, where $B(x) = (1 + x/L_0)B_0$. We assume a uniform plasma density, and allow for small but finite $k_{||}$. The dispersion relation for the *MS* wave is $k_x^2 - N_{10}^2\omega^2/c_A^2 = 0$ (N_{10}^2 is defined below). The phase-space diagram of the dispersion relations of the *MS* wave and three representative *CVK* modes is shown in Figure 2. We see that the *CVK* modes cross the two branches (incidence and reflection) of the *MS* wave at two separate places. Thus we break the whole process into several steps, each of which can be analyzed explicitly: (1) the incident *MS* wave crosses the resonance layer and excites the *CVK* modes; (2) the *CVK* modes propagate in k_x -space; (3) the *CVK* modes cross the reflection branch and convert part of their energy to the reflected *MS* wave; (4) the *CVK* modes continue to travel in k_x -space. But because they are kinetic in $v_{||}$, they contain collective modes, among them the weakly damped ion-Bernstein wave (*IBW*); it can leave the resonance layer, and be absorbed by electron Landau damping. The percentage of the energy that goes into the *IBW* defines the conversion coefficient. What is left in the *CVK* modes represents the direct absorption by the resonant ions.

We list our results here; a more detailed discussion follows.

$$T(\eta) = \exp(-2\eta), \quad R(\eta, \kappa) = \left(\frac{4\eta}{2 + \eta} \right)^2 |F(\eta, \kappa)|^2,$$

$$C(\eta, \kappa) = \frac{(2 - \eta)^2}{8\eta} R(\eta, \kappa), \quad A(\eta, \kappa) = 1 - T(\eta) - \frac{(2 + \eta)^2}{8\eta} R(\eta, \kappa),$$

where

$$\eta \equiv \frac{\pi}{4} (k_0 L_0) \frac{\omega_m^2}{\omega_M^2} \frac{(\gamma_M - 1)^2 N_{10}^2}{[1 + (\gamma_M - 1) N_{10}^2]^2 v_M^2}, \quad \kappa \equiv (k_0 L_0) \frac{k_{||} v_m}{\omega},$$

$$F(\eta, \kappa) \equiv \int_{-\infty}^{+\infty} d\xi G(\xi) \exp i \left(2\kappa\xi - \frac{\eta}{\pi} \int_{-\infty}^{\xi} d\xi' \int_{-\infty}^{+\infty} d\xi'' \frac{G(\xi'')}{\xi'' - \xi' + i0^+} \right),$$

$\gamma_M \equiv \omega/\Omega_M$, $N_{\parallel} \equiv k_{\parallel}c_A/\omega$, $N_{\perp 0}^2 \equiv [1 - (\gamma_M + 1)N_{\parallel}^2][1 + (\gamma_M - 1)N_{\parallel}^2]/[1 + (\gamma_M^2 - 1)N_{\parallel}^2]$, $k_0 \equiv N_{\perp 0}\omega/c_A$, v_m is the minority thermal speed, and $G(v_{\parallel}) \equiv g_m(v_{\parallel}) - \omega^{-1}k_{\parallel}v_m^2 g'_m(v_{\parallel})$, ($g_m(v_{\parallel})$ is the unperturbed minority v_{\parallel} -distribution).

The validity of the two-step mode-conversion approximation is that the k_x -width of each mode-conversion region, which is the inverse of the x -width of the resonance layer, is much smaller than the separation of the two branches. The width of the resonance layer comes from two contributions, one due to the mode coupling, and the other due to the Doppler shift caused by the thermal spread of v_{\parallel} . In the case of minority fundamental resonance, the coupling parameter δ is the ratio of minority to majority plasma frequency: $\delta = \omega_m/\omega_M$; hence the width of the resonance layer is given by $\omega - \Omega_m(x) - k_{\parallel}v_m \sim \omega\delta$, which leads to $\Delta x \sim L_0(\delta + k_{\parallel}v_m/\omega)$; thus the validity condition is $(\Delta x)^{-1} \ll 2k_0$.

The derivation of the equations can be outlined as follows. From Maxwell's equations we have $\nabla \times \nabla \times \mathbf{E}(\mathbf{x}) = 4\pi i\omega c^{-2} \mathbf{J}(\mathbf{x}; f)$, where the current \mathbf{J} depends functionally on the perturbed particle distribution function $f^{(1)}$. The familiar procedure is to solve the linearized Vlasov equation for $f^{(1)}$, by integrating along the unperturbed orbit, and obtain the current $\mathbf{J} = \chi \cdot \mathbf{E}$, where χ is the linear susceptibility. But it is well known that this introduces a (rapid varying) resonant denominator into χ . We can avoid this difficulty by omitting the resonant particles from χ ; they remain as external current on the right-hand side of the wave equation. The motion of these particles is governed by their Vlasov equation. After some algebra, and using congruent reduction [5] to eliminate the other components of the electric field, we obtain two coupled equations:

$$D_J(x; v_{\parallel}) J(\cdot; v_{\parallel}) = \frac{i}{\omega} E(\cdot), \quad D_E(k_x) E(\cdot) = -\frac{i}{\omega} \int dv_{\parallel} J(\cdot; v_{\parallel}),$$

where the dispersion functions are given by:

$$D_J(x; v_{\parallel}) = \frac{\omega - k_{\parallel}v_{\parallel} - \Omega_m(x)}{\omega\omega_m^2 G(v_{\parallel})},$$

$$D_E(k_x) = \frac{[1 + (\gamma_M^2 - 1)N_{\parallel}^2](k_0^2 - k_x^2)}{(\gamma_M^2 - 1)(\frac{1}{2}k_x^2 + k_{\parallel}^2) - (\gamma_M - 1)\omega^2/c^2},$$

Here we keep terms $O(k_{\perp}^2 \rho_m^2)$. $J = J_x - iJ_y = e_m \int dv_x dv_y (v_x - iv_y) f^{(1)}$ is the resonant current, and $E = E_x - iE_y$ is the component of electric field that rotates in the ion sense. In the dispersion function D_E we have used the cold plasma approximation, and ignored the x dependence of Alfvén speed c_A . Choosing either the x - or k_x -representation, one of the equations is algebraic. We can derive the following conservation law for the wave-action flux:

$$\frac{\partial}{\partial x} \left[(\dot{x})_E \frac{\partial D_E}{\partial \omega} E^2(x, k_x) \right] + \frac{\partial}{\partial k_x} \left[(\dot{k}_x)_J \int dv_{\parallel} \frac{\partial D_J}{\partial \omega} J^2(x, k_x; v_{\parallel}) \right] = 0,$$

where $E^2(x, k_x)$ and $J^2(x, k_x; v_{\parallel})$ are the Wigner functions [2] of E and J respectively. From this equation we can identify the wave-action flux associated with *CVK* modes.

The solution of these equations closely follows that in [1], so we just outline it here. For the two mode conversions (steps 1 and 3), we linearize the dispersion function about $\mp k_0$: $D_E(k_x) \approx (k_x \mp k_0)V_E$, where $V_E = 4k_0[1 - (\gamma_M - 1)N_{\parallel}^2]/(\gamma_M - 1)^2 N_{\perp 0}^4$. Then the coupled equations become a first order ODE, and can be solved easily. In between the two mode-conversion regions (step 2), we ignore the coupling. Thus *CVK* modes propagate according to a simple equation:

$$\left[x(v_{\parallel}) - i \frac{d}{dk_x} \right] J(k_x; v_{\parallel}) = 0,$$

where $x(v_{\parallel}) = -L_0 k_{\parallel} v_{\parallel} / \omega$. Below the second mode-conversion region, we cannot ignore the coupling. Eliminating E we obtain the following *CVK* equation:

$$\left[x(v_{\parallel}) - i \frac{d}{dk_x} \right] J(k_x; v_{\parallel}) = \frac{L_0 \omega_m^2}{\omega^2 D_E(k_x)} G(v_{\parallel}) \int dv'_{\parallel} J(k_x; v'_{\parallel}),$$

whose solution can be obtained by the method of [6]. However, in that calculation, the *IBW* can only appear as a superposition of *CVK* modes. In order to project out the *IBW*, we use the spectral deformation technique [7].

In conclusion let us highlight the main points involved in our analysis: (1) gyroresonant absorption as mode-conversion Case-van Kampen modes; (2) linear mode conversion in phase-space, and the closely related congruent reduction theory; (3) spectral deformation technique, which helps us to project out the ion-Bernstein wave. A more detailed account of this work is in preparation.

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